A DUAL MONTE CARLO APPROACH TO ESTIMATE MODEL UNCERTAINTY AND ITS APPLICATION TO THE RANGELAND HYDROLOGY AND EROSION MODEL

H. Wei, M. A. Nearing, J. J. Stone, D. D. Breshears

ABSTRACT. *Natural resources models serve as important tools to support decision making by predicting environmental indicators. All model predictions have uncertainty associated with them. Model predictive uncertainty, often expressed as the confidence interval around a model prediction value, may serve as important supplementary information for assisting decision making processes. In this article, we describe a new method called Dual Monte Carlo (DMC) to calculate model predictive uncertainty based on input parameter uncertainty. DMC uses two Monte Carlo sampling loops, which enable model users to not only calculate the model predictive uncertainty for selected input parameter sets of particular interest, but also to examine the predictive uncertainty as a function of model inputs across the full range of parameter space. We illustrate the application of DMC to the process-based, rainfall event-driven Rangeland Hydrology and Erosion Model (RHEM). The results demonstrate that DMC effectively generated model predictive uncertainty from input parameter uncertainty and provided information that could be useful for decision making. We found that for the model RHEM, the uncertainty intervals were strongly correlated to specific model input and output parameter values, yielding regression relationships (r^2 > 0.97) that enable accurate estimation of the uncertainty interval for any point in the input parameter space without the need to run the Monte Carlo simulations each time the model is used. Soil loss predictions and their associated uncertainty intervals for three example storms and three site conditions are used to illustrate how DMC can be a useful tool for directing decision making.*

Keywords. *Decision making, Model predictive uncertainty, RHEM, Soil erosion.*

The environmental indicators predicted by natural resources models are important for assisting decision making. However, a universal problem in applying models is how the model output deviates from the "true prediction." If the uncertainty associated with the model output (predictive uncertainty) can be quantified and propagated into model output, it may provide useful information for many model application purposes. For example, average annual soil loss values predicted by erosion models have served as a single indicator to help assess erosion risk and to choose conservation practices (Federal Register, 1997, 2004a, 2004b). However, a single soil loss value alone may not provide adequate information about the erosion state, and it can be difficult to justify decisions made on individual parcels of land based on a single erosion value. It is also difficult to know to what level of confidence in a given decision satisfactorily addresses an associated desired conservation goal, or more importantly, when a change in practice may cause the system to cross certain threshold states, thereby suggesting the need for use of a different conservation practice design. Knowing the level of uncertainty associated with the impact of a specific conservation plan may allow one to quantify the risk of failure of that practice as applied to a particular situation.

Model predictive uncertainty comes from multiple sources. The input parameter set that users provide is usually the representation of the average condition of a study site (for example, the average slope of a hillslope element). However, assigning a value to that representative variable inherently involves a certain degree of uncertainty, which will directly affect the level of uncertainty of the model prediction. Model predictive uncertainty also comes from model structure and internal parameter uncertainty (Chaves and Nearing, 1991). Structural uncertainty is associated with the inadequacy and the incompleteness of the model to represent the natural process. Internal parameter uncertainty refers to the coefficients set as constant values inside the model, as well as limitations with model equation structures. The variation of input parameter uncertainty at different sites and the complicated interaction between input parameters make the predictive uncertainty both site-specific and implicit.

There are different types of uncertainty analysis methods available for different purposes. For example, measurement uncertainty analyses, which often involve repetitive measurements and the so-called "first-order" or "*N*th-order" uncertainty estimation, were designed to determine measure- ment inaccuracies (Kline, 1985). Measurement uncertainty analyses have been regarded as an effective tool for evaluating and calibrating instruments and minimizing instrumentation costs (ASME/WAM, 1985). Generalized Likelihood Uncertainty Estimation (GLUE) is a method that was developed to evaluate model performance by looking at how the predicted value closes to the

Submitted for review in August 2007 as manuscript number SW 7170; approved for publication by the Soil & Water Division of ASABE in February 2008.

The authors are **Haiyan Wei, ASABE Member,** Research Specialist, **Mark A. Nearing,** Research Leader, and **Jeffry J. Stone,** Research Hydrologist, USDA-ARS Southwest Watershed Research Center, Tucson, Arizona; and **David D. Breshears,** Professor, School of Natural Resources, Institute for the Study of Planet Earth, and Department of Ecology and Evolutionary Biology, University of Arizona, Tucson, Arizona. **Corresponding author:** Haiyan Wei, USDA-ARS Southwest Watershed Research Center, Tucson, AZ 85719; phone: 520-670-6381, ext. 145; fax: 520-670-5550; e-mail: haiyan.wei@ars.usda.gov.

site-specific observation using an objective function or goodness-of-fit test (Freer and Beven, 1996; Brazier et al., 2000; Brazier et al., 2001; Aronica et al., 2002). GLUE is a useful tool for evaluating model performance for specific sites considering the model uncertainty from model structure and input parameters. Sampling-based uncertainty analysis is another method that can be used if one wants to know how a model responds to input over specified ranges (e.g., Breshears et al., 1989; Birchall and James, 1994; Cacuci and Ionescu-Bujor, 2004). This method usually first addresses the input uncertainty by assigning ranges of interest to each parameter, and then randomly samples different combinations of input parameter sets and calculates the outputs to examine uncertainty by looking at the range and the distribution of the outputs. However, there is currently no method that can both assess the site-specific uncertainty intervals and examine the model predictive uncertainty across the full range of the model parameter space.

The objective of this article was to develop a Dual Monte Carlo (DMC) uncertainty analysis method to calculate predictive uncertainty and then apply it to the Rangeland Hydrology and Erosion Model (RHEM) (Wei et al., 2007) as an example. The methodology of DMC is similar to the sampling-based uncertainty analysis mentioned above, but in our case we use two Monte Carlo sampling loops for assessing the entire ranges of all the input parameters. This was done for purposes of calculating the model uncertainty for specific sites and conditions, and to examine the uncertainty intervals as a function of model inputs across the full range of the parameter space. The difference between DMC and GLUE is that although DMC does not address the model structural uncertainty by comparing model predictions with site-specific observations as does GLUE, DMC does quantify the confidence of model predictions by calculating and comparing the model predictive uncertainty generated from input parameter uncertainty for different sites and different site conditions, thereby assisting decision making. This was demonstrated in this study by a comparison of uncertainty intervals associated with different erosion events.

Methods

The DMC approach starts with delineating a model input parameter space (denoted as I) by overlaying the full range of each selected input parameter (fig. 1). Then, the first Monte Carlo simulation is conducted to randomly select an individual point, x^0 (e.g., one input parameter set), from the parameter space I using the Latin hypercube (LH) sampling method (McKay et al., 1979). The model is executed, and the output value $y(x^0)$ is computed and saved. The second Monte Carlo simulation is conducted to calculate the uncertainty interval at x^0, a process that requires three steps: (1) assign the uncertainty to each input parameter at point x^0 by providing distribution information to all the input parameters; (2) build a local input space (denoted i) around point x^0 by combining the uncertainty of all the input parameters, then conduct the Monte Carlo simulation to randomly sample points from the local space i, and then calculate the corresponding output value for each (e.g., fig. 2 provides an example with two input parameters); and (3) extract the uncertainty intervals from the output distribution. For example, for a local space size associated with a 1000-run, the 1000 model output values form a distribution, of which the 950th largest value and the 50th largest value are the upper and lower uncertainty intervals at the confidence level of 95%. After finishing the second Monte Carlo simulation and saving the results, the initial Monte Carlo simulation is conducted again, and the process is repeated until sufficient points x^0 are selected for space I to be well covered. A uniform distribution is used for the first Monte Carlo simulation to ensure that parameter space I is well covered and sampled. The distribution type of the second Monte Carlo simulation depends on the nature of the input parameters, and it could vary for different parameters.

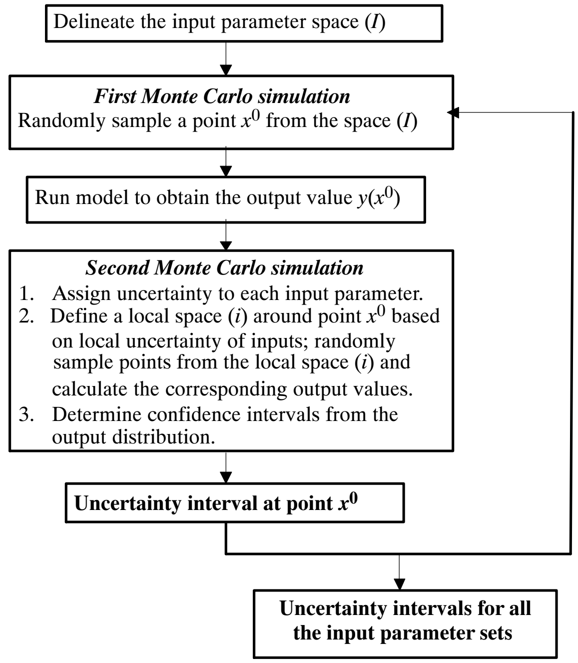

Figure 1. Flowchart of the methodology outlined in this article.

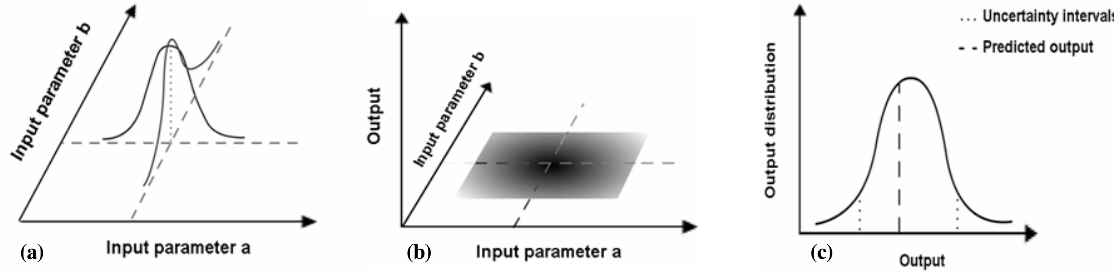

Figure 2. Steps for the second Monte Carlo simulation providing an example for two input parameters: (a) assign distribution for input parameters at x^0; (b) overlay the input parameter distributions to define a local space i around the point x^0, calculate output values for points sampled from i; and (c) extract the uncertainty intervals at certain confidence level from the distribution of output values.

RHEM and Model Inputs for Running DMC

RHEM is a newly conceptualized model that was adapted from relevant portions of the WEPP (Water Erosion Prediction Project) model (Flanagan and Nearing, 1995; Nearing et al., 1989; Laflen et al., 1997) and modified to specifically address rangeland conditions. It predicts the soil loss for rangelands based on the simulation of hydrology and erosion processes. A previous study was conducted to assess the sensitivity of the predicted erosion to the model input variables (Wei et al., 2007). RHEM provides a good case study for this article because soil loss rates on natural rangelands are generally low compared to other agricultural environments, and low erosion rates have been shown to be associated with high variability (Nearing et al., 1999). Quantitative estimations of uncertainty on the RHEM soil loss predictions will increase the ability of RHEM to direct decision making on choosing appropriate conservation practices.

To run the DMC, the output and input parameters of interest need to be determined, and the ranges and the uncertainty information for each input parameter must be assigned. In this study, the amount of soil erosion from the hillslope, *soiloss* (kg m^{-2}) was the targeted output parameter. Twelve required input parameters were selected for RHEM, and the full range of parameter values for each input variable was assigned (table 1). The sources of the ranges came from recommendations for rangeland applications in the WEPP model documentation (Flanagan and Nearing, 1995) and from WEPP rainfall simulation experimental databases (Elliot et al., 1989; Simanton et al., 1991; Laflen et al., 1991; Alberts et al., 1995).

Among the twelve input variables, *slp* and *sln* are the two parameters that represent the slope condition; *rain*, *xip*, and *dur* are rainfall parameters; k_e is the hydraulic conductivity; k_{ss}, k_r, and τ_c are erodibility factors; and f_r and f_e are friction factors. The particle size distribution factor (*psd*) was used in the model to build a lognormal distribution curve, from which five pairs of particle size and fraction data were obtained and passed to the transport capacity and deposition calculations (*psd* represents the mean value of the lognormal distribution). A standard deviation of 2.16 was calculated from the WEPP rangeland database and was used for the distribution of all types of sediments.

The inputs for storm and slope condition were considered as the driving force and given as constants; hence, no uncertainties were assigned to these parameters. The uncertainties of the remaining seven parameters were addressed using either the standard deviation (SD) or coefficient of variation (CV) (table 2). From the WEPP database, we compiled repetitive measurements of the input parameters to determine if the standard deviation or coefficient of variation (CV) could be used to describe the uncertainty. Such data on rangeland erosion parameters are rare, with the WEPP rangeland dataset being the most comprehensive data set of this type to date. Since the range of the values of individual parameters in the database used to characterize input distributions was similar to the domain of the input parameter space *I* (see tables 1 and 2), we believe that input uncertainty in table 2 can be applied to the entire parameter space *I*. The uncertainty of an input parameter may be dependent on the magnitude of the parameter. In our case, we found there was an exponential relationship between hydraulic conductivity k_e and the coefficient of variation of k_e (r^2 = 0.68) (fig. 3).

Table 1. Parameters and parameter ranges used in this study.

Parameter	Lower Bound	Upper Bound	Unit	Description
slp	3	30	%	Slope
slplen	10	100	m	Slope length
rain	20	120	mm	Rainfall volume
dur	0.5	2	h	Rainfall duration
xip	1	20	--	Rainfall intensity variable
k_{ss}	50	4600	--	Splash and sheet erosion coefficient
k_r	0.00004	0.003	s m^{-1}	Concentrated flow erosion coefficient
τ_c	0.0001	5.71	N m^{-2}	Critical shear stress
k_e	0.8	40	mm h^{-1}	Effective hydraulic conductivity
f_r	4.07	200	--	Friction factor for runoff
f_e	1.11	100	--	Friction factor for erosion
psd	-7	-1	--	Particle size distribution

Table 2. Input parameter uncertainty and the range of parameter values used to obtain the information on uncertainty.

Input Parameter	Min.[a]	Max.[a]	SD	CV (%)
k_{ss}	80	457		30.13
k_r (s m^{-1})	0.000048	0.0013		30.73
τ_c (N m^{-2})	0.20	2.23	0.8539	
k_e (mm h^{-1})	1.14	36.94		-43.11 log(k_e) + 1.38
psd	-6.56	-1.61		-7.81[b]
f_r	20.73	193.78		12.23
f_e	9.16	52.92		14.55

[a] Database size used to calculate input parameter uncertainty.
[b] The variation of *psd* was assumed equal to that of the primary sediment distribution.

Sampling Methods for the Monte Carlo Simulations

Because of the different purposes of the two Monte Carlo simulations, we used different sampling methods in each case. Latin hypercube (LH) sampling was used for the first Monte Carlo simulation to select random points from the uniformly distributed parameter space *I*. McKay et al. (1979) compared several sampling techniques, and found that the LH method has a number of desirable properties over others. One of the advantages of this method that makes it appropriate for this study is that LH ensures full coverage over the range of each variable so that all areas of the sample space

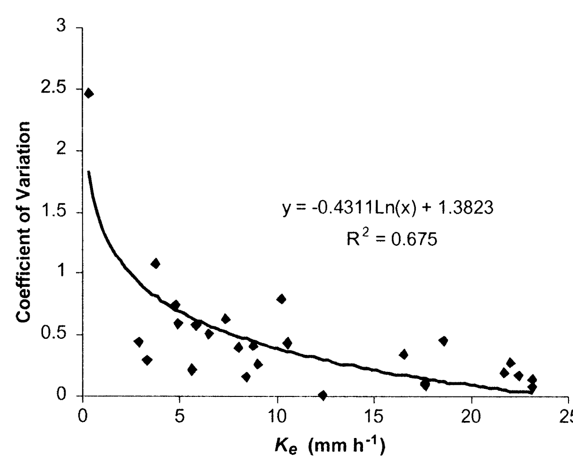

Figure 3. Example of how input parameter uncertainty was determined. A function was used to address uncertainty for parameter k_e, based on the relationship revealed in the data.

will be represented by the selected input values. The more points that are selected from the parameter space, the more the space will be densely covered and the more reliable the results will be. To ensure a well-covered parameter space, we compared the results from different sampling sizes. Statistical comparison of the relationship between uncertainty intervals and predicted output values showed there was no significant difference between the results from 200,000 points and 500,000 points for our simulations. Therefore, results from 200,000 points were used in this article. The purpose of the second Monte Carlo simulation was to randomly sample input parameter values and build a local space *i* based on the characteristics of the input distributions. For example, a normal distribution could be formed given the mean value and either the standard deviation or the coefficient of variation. The inverse normal distribution function was applied to generate the input parameter value for any given probability (0-1), with the probability based on random numbers generated from a random function in Visual Fortran. We sampled 1,000 points to build the space *i* for each point selected from space *I*.

Results and Discussions

For RHEM, the magnitude of the expected soil loss value and the associated uncertainty intervals based on a 95% confidence level at each point x^0 were highly related to the model-estimated soil loss $y(x^0)$ such that uncertainty increased with magnitude of soil loss (fig. 4). The expected soil loss value was computed as the mean of the 1000 values from the output distribution around each point x^0. On average, the mean values were within a few percent of the predicted val es. This means that for the model RHEM, the output distributions were not significantly skewed (either positively or negatively). Different results in this regard might be expected from different models. If the two values (expected and predicted) were significantly different, this would mean that the output distributions were highly skewed, in which case the model predictive uncertainty analysis would become even more important. The lower and upper limits of 95% confidence intervals for the estimated soil loss value (fig. 4) show that the uncertainties were highly dependent on the magnitude of the soil loss value.

The magnitude of the model predictive uncertainty depends not only on the input parameter values and the uncertainties associated with them, but also on the model structure, the sensitivity of the model output to the input parameters, and interactions between the model parameters. To assess whether our results of model uncertainty were realistic, we compared our overall results to the measured data from erosion plots. Erosion datasets for such a comparison are rare. Here we use data from Nearing et al. (1999) to examine the natural and measurement variability of soil loss. Nearing et al. (1999) collected measured soil loss data from replicated plot-pair for 2061 storms, 797 annual erosion measurements, and 53 multi-years, which represented 13 different site locations, each with different soil types. Nearing et al. (1999) calculated the mean value (M_m) and coefficient of variation (CV_m) from the replicated soil loss measurements, and found a linear relationship between the logarithm of CV_m and the logarithm of M_m: $\log(CV_m) = -0.306\log(M_m) - 0.442$ ($r^2 = 0.78$).

A comparative type of relationship with our predicted data from RHEM was obtained using the coefficient of variation (CV_p) and the expected value (M_p) obtained from the output distributions. The variance between replicates was related to the magnitude of the soil loss, with higher variation associated with smaller events. This was true for both the measured data and our model predictions (fig. 5). The regression line developed by Nearing et al. (1999) from the measured dataset fell within the range of our predicted data, which indicated that DMC gave realistic quantifications of the model uncertainty. Although our predicted relationship differed slightly in slope from the regression developed by Nearing et al. (1999), this difference is minor compared to the overall variation in the observed data and could be due to several factors: our model assessment had more sampling points and did not consider the input uncertainty on the slope and rainfall parameters, and our comparison does not include uncertainty in model structure.

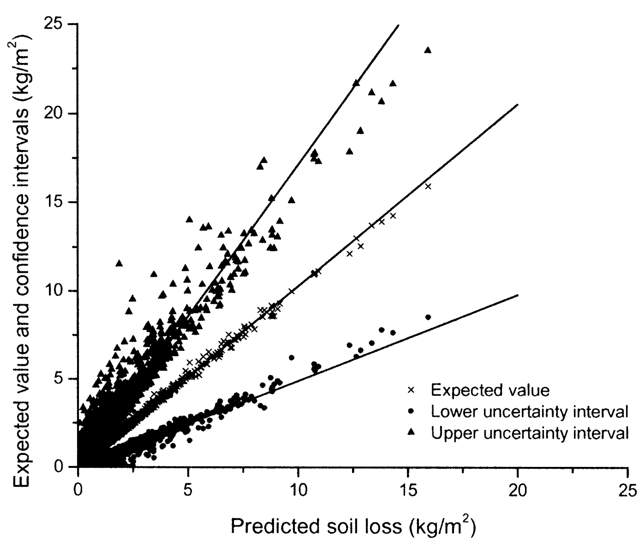

Figure 4. Expected prediction and uncertainty intervals vs. the predicted value.

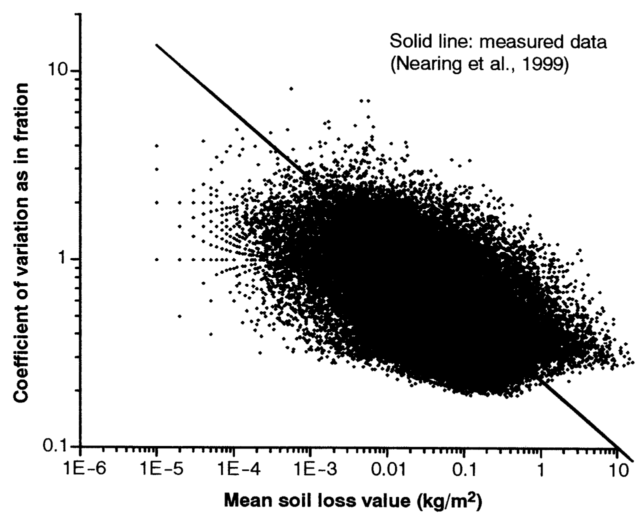

Figure 5. Coefficient of variation (CV) for predicted soil loss from the output distribution at a point vs. the expected predicted soil loss value. The corresponding relationship developed from measured data (Nearing, 1999) is also shown.

The high correlations between the predicted soil loss value and uncertainty intervals (r^2 of 0.95 and 0.97 in fig. 4) indicate that the relationships can be used to quantify the model uncertainty. Furthermore, we found that the model uncertainty intervals were also highly related to the output parameter, to runoff depth, and to some input parameter values such as rainfall amount and saturated hydraulic conductivity. We developed two regression functions that even had greater r^2 values for the upper and lower uncertainty intervals at 95% confidence level:

Upper interval (r^2 = 0.97)

$$= 0.01 + 2.2\,soiloss + 0.004(soiloss*rain) - 0.02(soiloss*k_e) - 13.7(soiloss*runoff)$$

Lower interval (r^2 = 0.98)

$$= -0.002 + 0.39\,soiloss - 0.001(soiloss*rain) + 0.004(soiloss*k_e) + 3.3(soiloss*runoff)$$

The advantage of these equations is that they allow the model developers to provide to the user an estimate of the confidence range for a given model output without the need to run a full Monte Carlo simulation around the user's point of interest (x^0) each time the model is used. We used the stepwise multiple variable linear regression in SAS and selected equations with the highest r^2 to generate the two above functions. The variables in the equations were selected from all the input parameter, the product of input parameters, the output runoff depth (*runoff*, m), and the output soil loss rate (*soiloss*, kg m^{-2}). The predicted uncertainty intervals from the equations were very close to those calculated from DMC (fig. 6).

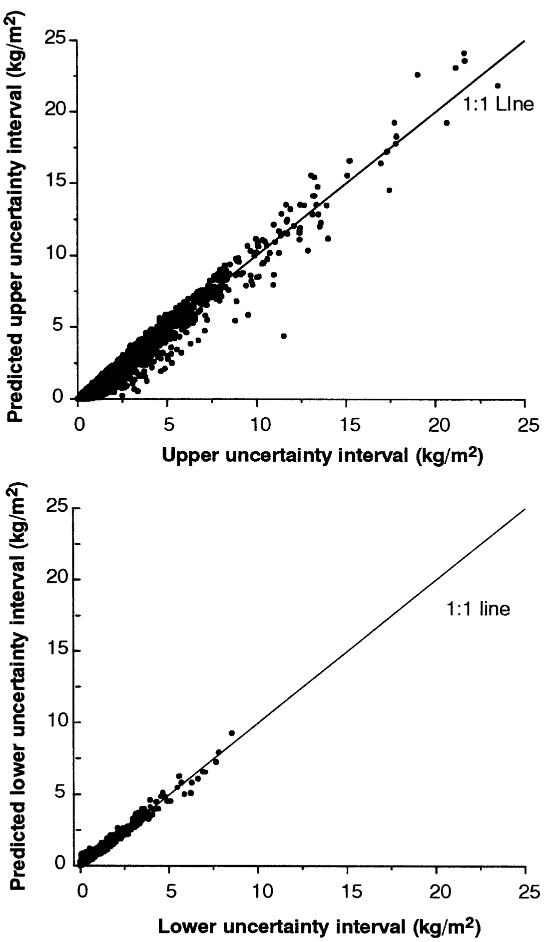

Figure 6. Predicted uncertainty intervals from the regression equations vs. uncertainty intervals calculated from DMC.

Application

To show how the predictive uncertainty can be used to assist decisions relative to natural resources management, we applied our results to a 0.18 ha shrub-dominated watershed located in the USDA-ARS Walnut Gulch Experimental Watershed in southeastern Arizona. We calculated the soil loss from RHEM and the associated uncertainty from the equations derived in the previous section for three different sizes of storms (table 3: 1-year, 25-year, and 100-year return rainfall amount) and three different site conditions (table 4: current, moderately degraded, and severely degraded). The values of hydraulic conductivity and soil erodibility coefficients for the current condition in table 4 were obtained from the rangeland WEPP rainfall simulation dataset, and we arbitrarily decreased k_e and τ_c and increased k_{ss} and k_r by different amounts to simulate the different site conditions, since poor conditions could be expected to relate to low hydraulic conductivity due to the soil compaction, and to higher soil erodibility due to less vegetation coverage.

The magnitude of the uncertainty intervals varied with the size of storm and site conditions (fig. 7). To evaluate the erosional risk, we referred to Rollins' (1982) general estimation of the T factor (soil loss tolerance rate) for rangeland as a reference level. The T factor is defined as the predetermined value of soil loss below which there will be limited effect of soil loss on the fertility and the productivity of soil in an economic sense. Rollins (1982) proposed a T factor of 1 ton acre^{-1} year^{-1} for rangelands and pointed out that rangelands cannot tolerate the same soil loss as croplands due to the low soil formation rate that occurs in rangelands. Results indicated that the absolute uncertainty intervals increased as the size of the storm increased and as the degree of assumed land degradation increased (fig. 7). Furthermore, the upper uncertainty intervals for the moderately degraded and severely degraded conditions from the 25-year and 100-year rainfall events were quite close to the yearly T value. Considering the high predictive uncertainty associated with these two erosion events, decision makers may want to choose conversation practices stronger than the practices traditionally selected based on the model-predicted soil loss values.

Table 3. Storm input parameters.

Frequency (year)	1	25	100
Duration (h)	0.5	1	3
Rainfall (mm)	17.78	51.31	76.71

Table 4. Hydrological and erosion parameters for different conditions.

Parameter	Current Condition	Moderately Degraded	Severely Degraded
k_e (mm h^{-1})	28.7	28.7 × 0.5 = 14.35	28.7 × 0.05 =1.44
k_{ss}	435	435 × 3 = 1305	435 × 6 = 2610
k_r (s m^{-1})	0	0.0001	0.001
τ_c (N m^{-2})	0	1.20	0.5

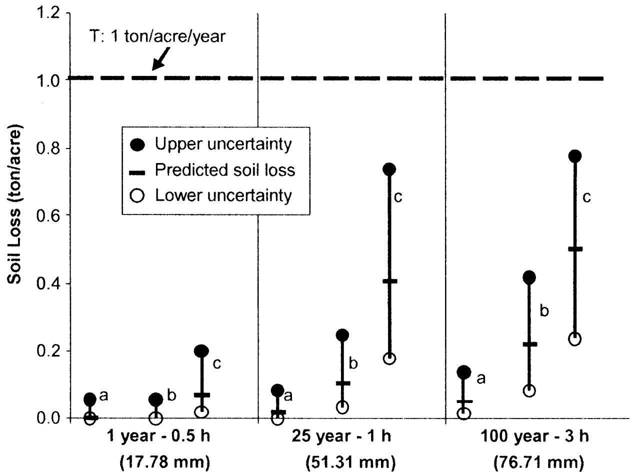

Figure 7. Predictive uncertainty for three different storms and three different site conditions. The abscissa is divided into three sections for three storm sizes (17.78, 51.31, and 76.71 mm). Site conditions are indicated by letters a, b, and c, where a is the current condition, b is the moderately degraded condition, and c is the severely degraded condition. For each event, the predicted soil loss rate and the upper and lower uncertainty intervals are connected by a vertical line. The soil loss tolerance value (1 ton acre^{-1} year^{-1}; the horizontal dashed line) is also given in the figure as a reference to evaluate the erosional risk.

CONCLUSIONS

A Dual Monte Carlo (DMC) approach was developed to calculate the predictive uncertainty for any selected model input parameter set and to simultaneously examine the uncertainty intervals as a function of the output and input parameter values. A case study using the Rangeland Hydrology and Erosion Model (RHEM) describes the framework of DMC. A comparison of the variations of RHEM-predicted soil loss from the DMC approach with the measured soil loss variations published by Nearing et al. (1999) demonstrated that DMC provided realistic estimation of model uncertainty for RHEM. The calculated model predictive uncertainty of RHEM was used to evaluate the erosion risk for different scenarios. The results indicated that model uncertainty increased with increased storm size and increased degradation levels, and therefore decision makers may want to choose different conservation plans than what a single predicted soil loss value would suggest. In summary, the DMC approach can be used to quantify model predictive uncertainty, which provides supplementary information and increases the model's ability for assisting the decision making process.

REFERENCES

Alberts, E., M. A. Nearing, M. A. Weltz, L. M. Risse, F. B. Pierson, X. C. Zhang, J. M. Laflen, and J. R. Simanton. 1995. Chapter 7: Soil component. In *USDA Water Erosion Prediction Project: Hillslope Profile and Watershed Model Documentation.* D. C. Flanagan and M. A. Nearing, eds. NSERL Report No. 10. West Lafayette, Ind.: USDA-ARS National Soil Erosion Research Laboratory.

Aronica, G., D. Bates, and M. Horritt. 2002. Assessing the uncertainty in distributed model predictions using observed binary pattern information within GLUE. *Hydrol. Processes* 16(10): 2001-2016.

ASME/WAM. 1985. The 1983 ASME/WAM symposium on uncertainty analysis: Theory and practice of experimental control. *J. Fluids Eng.* 107(2): 153-182.

Birchall, A., and A. James. 1994. Uncertainty analysis of the effective dose per unit exposure from radon progeny and implications for ICRP risk-weighting factors. *Radiation Protection Dosimetry* 53: 133-140.

Brazier, R., K. Beven, J. Freer, and J. Rowan. 2000. Equifinality and uncertainty in physically based soil erosion models: Application of the GLUE methodology to WEPP – the Water Erosion Prediction Project – for sites in the UK and USA. *Earth Surface Process and Landforms* 25(8): 825-845.

Brazier, R., K. Beven, S. Anthony, and J. Rowan. 2001. Implications of model uncertainty for the mapping of hillslope-scale soil erosion predictions. *Earth Surface Processes and Landforms* 26(12): 1333-1352.

Breshears, D. D., T. B. Kirchner, M. D. Otis, and F. W. Whicker. 1989. Uncertainty in predictions of fallout radionuclides in foods and of subsequent ingestion. *Health Physics* 57(6): 943-953.

Cacuci, D., and M. Ionescu-Bujor. 2004. A comparative review of sensitivity and uncertainty analysis of large-scale systems: II. Statistical methods. *Nuclear Sci. and Eng.* 147(3): 189-203.

Chaves, H., and M. A. Nearing. 1991. Uncertainty analysis of the WEPP soil-erosion model. *Trans. ASAE* 34(6): 2437-2445.

Elliot, W. J., A. M. Liebenow, J. M. Laflen, and K. D. Kohl. 1989. A compendium of soil erodibility data from WEPP cropland soil field erodibility experiments 1987 & 88. NSERL Report No. 3. West Lafayette, Ind.: USDA-ARS National Soil Erosion Research Laboratory.

Federal Register. 1997. USDA Commodity Credit Corporation, FSA. 7 CFR Parts 704 and 1410. *Conservation Reserve Program* 62(33): 7601-7635.

Federal Register. 2004a. USDA Commodity Credit Corporation, NRCS. 7 CFR Part 1469. *Conservation Security Program* 69(118): 34502-34532.

Federal Register. 2004b. USDA Commodity Credit Corporation, NRCS. 7 CFR Part 491. *Farm and Ranch Lands Protection Program* 71(144): 42567-42572.

Flanagan, D. C., and M. A. Nearing. 1995. USDA-Water Erosion Prediction Project: Hillslope profile and watershed model documentation. NSERL Report No. 10. West Lafayette, Ind.: USDA-ARS National Soil Erosion Research Laboratory.

Freer, J., and K. Beven. 1996 Bayesian estimation of uncertainty in runoff prediction and the value of data: An application of the GLUE approach. *Water Resources Res.* 32(7): 2161-2173.

Kline, S. J. 1985. The purpose of uncertainty analysis. *J. Fluids Eng.* 107(2): 153-160.

Laflen, J. M., W. J. Elliot, R. Simanton, S. Holzhey, and K. D. Kohl. 1991. WEPP soil erodibility experiments for rangeland and cropland soils. *J. Soil and Water Cons.* 46(1): 39-44.

Laflen, J. M., W. J. Elliot, D. C. Flanagan, C. R. Meyer, and M. A. Nearing. 1997. WEPP: Predicting water erosion using a process-based model. *J. Soil and Water Cons.* 52(2): 96-102.

McKay, M. D., W. J. Conover, and R. J. Beckman. 1979. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics* 21: 239-245.

Nearing, M. A., G. R. Foster, L. J. Lane, and S. C. Finkner. 1989. A process-based soil erosion model for USDA-Water Erosion Prediction Project technology. *Trans. ASAE* 32(5): 1587-1593.

Nearing, M. A., G. Govers, and D. Norton. 1999. Variability in soil erosion data from replicated plots. *SSSA J.* 63(6): 1829-1835.

Rollins, B. M. 1982. "T" factor on rangelands. In *Proc. Workshop on Estimating Erosion and Sediment Yield on Rangelands*. Oakland, Cal.: USDA-ARS, Western Region.

Simanton, J. R., M. A. Weltz, and H. D. Larson. 1991. Rangeland experiments to parameterize the Water Erosion Prediction Project model: Vegetation canopy cover effects. *J. Range Mgmt.* 44(3): 276-281.

Wei, H., M. A. Nearing, and J. J. Stone. 2007. A comprehensive sensitivity analysis framework for model evaluation and improvement using a case study of the rangeland hydrology and erosion model. *Trans. ASABE* 50(3): 945-953.